Incoherent Regeneration at CPLEAR

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Abstract

The regeneration correction applied in the analysis of neutral kaon decay rates at CPLEAR so far only takes into account coherent effects ("transmission regeneration"). The purpose of this note is to investigate the effect of incoherent ("diffraction") regeneration in CPLEAR measurements of CP violation as well as in the regeneration measurement itself by computing a quantitative estimate of the associated change in the observed decay rate asymmetries.

1 Introduction

1.1 Three classes of regeneration

According to the different ways of kaon scattering in condensed matter one generally distinguishes three classes of regeneration [1]:

Coherent (transmission) regeneration: elastic scattering by nuclei in the forward direction where the scatterers act coherently over an extended region of several centimetres length. The coherence condition yields $\theta \leq 10^{-7}$ for the limiting scattering angle.

Incoherent (diffraction) regeneration: elastic diffractive scattering on individual nuclei in any direction (including forward). Incoherent addition of intensities from different nuclei, but coherent action of the nucleons inside the nucleus.

Inelastic regeneration: inelastic scattering with large momentum transfer. Excitation or disintegration of the nucleus.

All three types of regeneration have been observed, for the first time by R. H. Good et al.[2].

1.2 Incoherent regeneration at CPLEAR

In classical regeneration experiments measuring the intensity of $\mathrm{K_S}$ regenerated from a pure $\mathrm{K_L}$ beam, incoherent regenerative scattering in the forward direction appears as a significant background for the coherent process. In the CPLEAR experimental setup, however, the situation is quite different: the primary observable at CPLEAR, the asymmetry between twopionic decay rates of initially pure $\mathrm{K^0}$ and $\overline{\mathrm{K}}^0$[3]

$$A_{+-}(\tau) = \frac{\overline{R}(\tau) - R(\tau)}{\overline{R}(\tau) + R(\tau)} \tag{1}$$

can be very sensitive to coherent regeneration – depending on the position of the regenerating material – because of interference of the inherent $\mathrm{K_S}$ amplitude of the incoming neutral kaon with the regenerated $\mathrm{K_S}$ amplitude and similarly, but less relevant, for $\mathrm{K_L}$. By definition, the coherently regenerated amplitude alone can cause such an interference whereas incoherently regenerated $\mathrm{K_S}$ only give rise to some "background" of two pion decays after the regenerator within the angular acceptance of the detector. The relative number of such additional decays is small for regeneration not too far from the production point and would be equal for $\mathrm{K^0}$ and

$\overline{\mathrm{K}}^0$ if there was no CP violation. Therefore, we expect the effect of incoherent regeneration in asymmetry measurements to be quite small in comparison with coherent regeneration, especially at relatively short lifetimes.

However, with the high precision attained by the CPLEAR experiment it is nevertheless desirable to have a quantitative estimate of the size of the effect. To this end we develop the phenomenology of diffraction regeneration in the following section and apply it in a numerical computation in Sec. 4.

We definitely need not worry about inelastic regeneration as the corresponding cross sections at energies available at CPLEAR are one or two orders of magnitude smaller than their elastic counterparts.

2 Phenomenology of incoherent regeneration

We follow the analysis of M. L. Good [4] but use a somewhat modernized notation [5]. A neutral kaon state is described as a linear combination of either K^0 and $\overline{\mathrm{K}}^0$ or $\mathrm{K_L}$ and $\mathrm{K_S}$:

$$\psi(\tau) = \alpha_{\mathrm{L}}(\tau)\mathrm{K_L} + \alpha_{\mathrm{S}}(\tau)\mathrm{K_S} = \alpha(\tau)\mathrm{K}^0 + \bar{\alpha}(\tau)\overline{\mathrm{K}}^0$$

where τ denotes the particle's proper time. In a regenerating medium the amplitudes for the coherent wave (the "unscattered beam" in Good's language) are given by Good's equations, which take the following form in the kaon's rest frame:

$$\mathrm{i}\frac{\mathrm{d}\alpha_{\mathrm{L}}}{\mathrm{d}\tau} = \lambda_{\mathrm{L}}\alpha_{\mathrm{L}} + \left(\frac{\varkappa+\bar{\varkappa}}{2}\right)\alpha_{\mathrm{L}} + \left(\frac{\varkappa-\bar{\varkappa}}{2}\right)\alpha_{\mathrm{S}}, \tag{2a}$$

$$\mathrm{i}\frac{\mathrm{d}\alpha_{\mathrm{S}}}{\mathrm{d}\tau} = \lambda_{\mathrm{S}}\alpha_{\mathrm{S}} + \left(\frac{\varkappa+\bar{\varkappa}}{2}\right)\alpha_{\mathrm{S}} + \left(\frac{\varkappa-\bar{\varkappa}}{2}\right)\alpha_{\mathrm{L}}, \tag{2b}$$

where λ_{L} and λ_{S} are the familiar eigenvalues of the effective Hamiltonian and $\varkappa$ and $\bar{\varkappa}$ are given by

$$\varkappa = -\frac{2\pi N}{m} f(0) \qquad \text{and} \qquad \bar{\varkappa} = -\frac{2\pi N}{m}\bar{f}(0)$$

with $f(0)$ and $\bar{f}(0)$ the nuclear forward scattering amplitudes for K^0 and $\overline{\mathrm{K}}^0$, respectively, N the scattering centre density and m the mass of the kaon state. With the solution of (2),

$$\alpha_{\mathrm{L}}(\tau) = \mathrm{e}^{-\mathrm{i}\Sigma\cdot\tau}\left[\alpha_{\mathrm{L}}(0)\cos(\Omega\tau) - \frac{\mathrm{i}}{2\Omega}(\Delta\lambda\alpha_{\mathrm{L}}(0) + \Delta\varkappa\alpha_{\mathrm{S}}(0))\sin(\Omega\tau)\right], \tag{3a}$$

$$\alpha_{\mathrm{S}}(\tau) = \mathrm{e}^{-\mathrm{i}\Sigma\cdot\tau}\left[\alpha_{\mathrm{S}}(0)\cos(\Omega\tau) + \frac{\mathrm{i}}{2\Omega}(\Delta\lambda\alpha_{\mathrm{S}}(0) - \Delta\varkappa\alpha_{\mathrm{L}}(0))\sin(\Omega\tau)\right] \tag{3b}$$

where

$$\Omega = \tfrac{1}{2}\sqrt{\Delta\lambda^2 + \Delta\varkappa^2} \quad \text{and} \quad \Sigma = \tfrac{1}{2}(\lambda_{\mathrm{L}} + \lambda_{\mathrm{S}} + \varkappa + \bar{\varkappa}),$$

it is straightforward to calculate the amplitudes α_{L} and α_{S} at any position inside the regenerator. The probability for having a $\mathrm{K_S}$ is then $|\alpha_{\mathrm{S}}|^2$ and the two pion decay rate is proportional to $|\alpha_{\mathrm{S}} + \eta\alpha_{\mathrm{L}}|^2$.

For scattering at an angle θ we define the corresponding scattering amplitudes $f(\theta)$ and $\bar{f}(\theta)$. Since in our phase convention[1] $\mathrm{K_S} = \frac{1}{\sqrt{2}}(\mathrm{K}^0 - \overline{\mathrm{K}}^0)$ the differential cross section for the scattering of the state ψ into the final state $\mathrm{K_S}$ is given by

$$\frac{\mathrm{d}\sigma_{\psi\to\mathrm{K_S}}}{\mathrm{d}\Omega}(\theta) = \left|\frac{\alpha f(\theta) - \bar{\alpha}\bar{f}(\theta)}{\sqrt{2}}\right|^2 = \left|\frac{\alpha_{\mathrm{L}}[f(\theta) - \bar{f}(\theta)] + \alpha_{\mathrm{S}}[f(\theta) + \bar{f}(\theta)]}{2}\right|^2 \tag{4a}$$

[1] CP violation may be ignored at this point as it cancels anyway in the right hand side of Eq. (4).

where the αs are evaluated at the scattering point according to (2). By analogy we also get

$$\frac{d\sigma_{\psi\to K_L}}{d\Omega}(\theta) = \left|\frac{\alpha f(\theta)+\bar{\alpha}\bar{f}(\theta)}{\sqrt{2}}\right|^2 = \left|\frac{\alpha_L[f(\theta)+\bar{f}(\theta)]+\alpha_S[f(\theta)-\bar{f}(\theta)]}{2}\right|^2. \tag{4b}$$

To obtain the number of K_S (K_L) scattered at θ into a solid angle $d\Omega$ and originating from a regenerator slice of thickness dx at position x we multiply by Ndx, the area density of scattering centres in dx:

$$dn_S(x,\theta) = \frac{1}{4}\left|\alpha_L(x)[f(\theta)-\bar{f}(\theta)]+\alpha_S(x)[f(\theta)+\bar{f}(\theta)]\right|^2 N dx d\Omega$$
$$dn_L(x,\theta) = \frac{1}{4}\left|\alpha_L(x)[f(\theta)+\bar{f}(\theta)]+\alpha_S(x)[f(\theta)-\bar{f}(\theta)]\right|^2 N dx d\Omega$$

We integrate over the solid angle $d\Omega = 2\pi\theta d\theta$ and replace distance by proper time according to $dx = \gamma v d\tau$ to arrive at

$$dn_S(\tau_{sc}) = \frac{\pi}{2}\gamma v N d\tau_{sc}\int d\theta\,\theta A(\theta)\left|\alpha_L(\tau_{sc})[f(\theta)-\bar{f}(\theta)]+\alpha_S(\tau_{sc})[f(\theta)+\bar{f}(\theta)]\right|^2$$
$$dn_L(\tau_{sc}) = \frac{\pi}{2}\gamma v N d\tau_{sc}\int d\theta\,\theta A(\theta)\left|\alpha_L(\tau_{sc})[f(\theta)+\bar{f}(\theta)]+\alpha_S(\tau_{sc})[f(\theta)-\bar{f}(\theta)]\right|^2$$

where τ_{sc} stands for the time of the scattering and $A(\theta)$ is the angular acceptance of the detector. Any K_S or K_L emerging from such a process is subject to further incoherent or coherent scattering as well as absorption or decay. Due to the modest size and density of the regenerator installed in the CPLEAR detector (2.5 cm × 1.85 g/cm^3 carbon) we may neglect multiple scattering (i.e. further incoherent regeneration)[6]. This applies all the more to scattering in the detector walls, of course. The remaining three processes are well described in the framework of coherent regeneration.

3 Contribution to the measured 2π decay rate

Let us assume, as in [5], that the kaons perpendicularly penetrate a regenerator of finite thickness L. For a given momentum $p = \gamma v m$ they will enter the medium at proper time τ_1 and leave it again at τ_2. We are interested in the number of additional two pion decays at some proper time $\tau \geq \tau_2$ caused by incoherent regeneration. This contribution to two pion decays is given by the number of particles times their twopionic decay rate. For K_S incoherently produced at τ_{sc} for instance, the (normalized) contribution at τ is

$$dn_S(\tau_{sc})R_S^{2\pi}(\tau_{sc},\tau_2,\tau)$$

where

$$R_S^{2\pi}(\tau_{sc},\tau_2,\tau) = \left|\alpha_S^{(S)}(\tau)+\eta\alpha_L^{(S)}(\tau)\right|^2$$

and the superscripts of the amplitudes α indicate that they are evaluated according to (3) for a K_S at the scattering time τ_{sc}: Starting from

$$\alpha_S^{(S)}(\tau_{sc}) = 1 \quad \text{and} \quad \alpha_L^{(S)}(\tau_{sc}) = 0$$

we get, if we neglect the additional path length inside the medium due to the scattering angle,

$$\alpha_S^{(S)}(\tau_2) = e^{-i\Sigma\cdot(\tau_2-\tau_{sc})}\left[\cos(\Omega(\tau_2-\tau_{sc}))+\frac{i\Delta\lambda}{2\Omega}\sin(\Omega(\tau_2-\tau_{sc}))\right],$$
$$\alpha_L^{(S)}(\tau_2) = -e^{-i\Sigma\cdot(\tau_2-\tau_{sc})}\left[\frac{i\Delta\varkappa}{2\Omega}\sin(\Omega(\tau_2-\tau_{sc}))\right]$$

and simply

$$\alpha_S^{(S)}(\tau) = e^{-i\lambda_S(\tau-\tau_2)}\alpha_S^{(S)}(\tau_2),$$
$$\alpha_L^{(S)}(\tau) = e^{-i\lambda_L(\tau-\tau_2)}\alpha_L^{(S)}(\tau_2)$$

for the trajectory after the regenerator. To avoid any confusion in the final result we reserve the Greek letter α for the amplitudes of the unscattered beam and rename the amplitudes of the scattered particles in the following manner:

$$a_{LL} := \alpha_L^{(L)} \qquad a_{LS} := \alpha_L^{(S)}$$
$$a_{SL} := \alpha_S^{(L)} \qquad a_{SS} := \alpha_S^{(S)}$$

The total number of two pion decays from incoherently regenerated kaons is given by the sum of the contributions from K_S and K_L integrated over the thickness of the regenerator. The final result is

$$R_{inc}^{2\pi}(\tau) = \frac{\pi}{2}\gamma v N \int_{\tau_1}^{\tau_2} d\tau_{sc}\left(R_L^{2\pi}(\tau_{sc},\tau_2,\tau) I_L(\tau_1,\tau_{sc}) + R_S^{2\pi}(\tau_{sc},\tau_2,\tau) I_S(\tau_1,\tau_{sc})\right) \tag{5}$$

with

$$R_L^{2\pi}(\tau_{sc},\tau_2,\tau) = |a_{SL}(\tau_{sc},\tau_2,\tau) + \eta a_{LL}(\tau_{sc},\tau_2,\tau)|^2,$$
$$R_S^{2\pi}(\tau_{sc},\tau_2,\tau) = |a_{SS}(\tau_{sc},\tau_2,\tau) + \eta a_{LS}(\tau_{sc},\tau_2,\tau)|^2,$$

$$I_{L,S}(\tau_1,\tau_{sc}) = \int d\theta\, \theta A(\theta) \left|\alpha_{L,S}(\tau_{sc})[f(\theta)+\bar f(\theta)] + \alpha_{S,L}(\tau_{sc})[f(\theta)-\bar f(\theta)]\right|^2 \tag{6}$$

and

$$a_{LL}(\tau_{sc},\tau_2,\tau) = e^{-i\lambda_L(\tau-\tau_2)} e^{-i\Sigma\cdot(\tau_2-\tau_{sc})}\left[\cos(\Omega(\tau_2-\tau_{sc})) - \frac{i\Delta\lambda}{2\Omega}\sin(\Omega(\tau_2-\tau_{sc}))\right],$$
$$a_{LS}(\tau_{sc},\tau_2,\tau) = -e^{-i\lambda_L(\tau-\tau_2)} e^{-i\Sigma\cdot(\tau_2-\tau_{sc})}\frac{i\Delta\varkappa}{2\Omega}\sin(\Omega(\tau_2-\tau_{sc})),$$
$$a_{SL}(\tau_{sc},\tau_2,\tau) = -e^{-i\lambda_S(\tau-\tau_2)} e^{-i\Sigma\cdot(\tau_2-\tau_{sc})}\frac{i\Delta\varkappa}{2\Omega}\sin(\Omega(\tau_2-\tau_{sc})),$$
$$a_{SS}(\tau_{sc},\tau_2,\tau) = e^{-i\lambda_S(\tau-\tau_2)} e^{-i\Sigma\cdot(\tau_2-\tau_{sc})}\left[\cos(\Omega(\tau_2-\tau_{sc})) + \frac{i\Delta\lambda}{2\Omega}\sin(\Omega(\tau_2-\tau_{sc}))\right].$$

$\alpha_L(\tau_{sc})$ and $\alpha_S(\tau_{sc})$ are the coherently regenerated (i.e. unscattered) amplitudes at the time of the scattering τ_{sc}, recursively calculated with (3) as

$$\alpha_L(\tau_{sc}) = e^{-i\Sigma\cdot(\tau_{sc}-\tau_1)}\left[\alpha_L(\tau_1)\cos(\Omega(\tau_{sc}-\tau_1)) - \frac{i}{2\Omega}(\Delta\lambda\alpha_L(\tau_1) + \Delta\varkappa\alpha_S(\tau_1))\sin(\Omega(\tau_{sc}-\tau_1))\right],$$
$$\alpha_S(\tau_{sc}) = e^{-i\Sigma\cdot(\tau_{sc}-\tau_1)}\left[\alpha_S(\tau_1)\cos(\Omega(\tau_{sc}-\tau_1)) + \frac{i}{2\Omega}(\Delta\lambda\alpha_S(\tau_1) - \Delta\varkappa\alpha_L(\tau_1))\sin(\Omega(\tau_{sc}-\tau_1))\right];$$

$$\alpha_L(\tau_1) = e^{-i\lambda_L\tau_1}\alpha_L(0),$$
$$\alpha_S(\tau_1) = e^{-i\lambda_S\tau_1}\alpha_S(0)$$

from the initial amplitudes

$$\alpha_L(0) = \alpha_S(0) = \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1+\epsilon)} \approx \frac{1-\epsilon}{\sqrt{2}} \quad \text{for} \quad K^0$$

and

$$\alpha_{\rm L}(0) = -\alpha_{\rm S}(0) = \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1-\epsilon)} \approx \frac{1+\epsilon}{\sqrt{2}} \quad \text{for} \quad \overline{\rm K}^0.$$

$R^{2\pi}_{\rm inc}(\tau)$ is to be added to the decay rate after coherent regeneration, $R^{2\pi}_{\rm coh}(\tau)$, to obtain the observed decay rate:

$$R^{2\pi}_{\rm obs}(\tau) = R^{2\pi}_{\rm coh}(\tau) + R^{2\pi}_{\rm inc}(\tau) \tag{7}$$

For all practical purposes the contribution of $\rm K_L$ to the 2π decay rate may safely be neglected and (5) reduces to

$$R^{2\pi}_{\rm inc}(\tau) = \frac{\pi}{2}\gamma v N \int_{\tau_1}^{\tau_2} {\rm d}\tau_{\rm sc} |a_{\rm SS}(\tau_{\rm sc},\tau_2,\tau)|^2 I_{\rm S}(\tau_1,\tau_{\rm sc}) \tag{8}$$

4 Numerical computation for $\pi^+\pi^-$ decays at CPLEAR

In order to carry out the integration (8) we must know the angular variation of the scattering amplitudes $f(\theta)$, $\bar{f}(\theta)$ and the detector acceptance $A(\theta)$.

$f(\theta)$ and $\bar{f}(\theta)$ can be well reproduced by an optical model calculation which results in an integration over a weighted Bessel function. The weighing function to be applied is not the same for $\rm K^0$ and $\overline{\rm K}^0$ reflecting the different interaction length of the two states inside the nucleus and therefore $f(\theta)$ and $\bar{f}(\theta)$ have somewhat different slopes [7]. For our estimate, however, we do not take into account this rather small difference [9] and assume furthermore that only the magnitudes of the amplitudes are affected by the angular variation:

$$\frac{\bar{f}(\theta)}{\bar{f}(0)} \approx \frac{f(\theta)}{f(0)} \approx \frac{|f(\theta)|}{|f(0)|} =: F(\theta),$$

Thus the θ integrations (6) simplify to

$$I_{\rm L,S}(\tau_1,\tau_{\rm sc}) = \left|\alpha_{\rm L,S}(\tau_{\rm sc})[f(0)+\bar{f}(0)] + \alpha_{\rm S,L}(\tau_{\rm sc})[f(0)-\bar{f}(0)]\right|^2 I_\theta$$

with

$$I_\theta = \int {\rm d}\theta\, \theta A(\theta)|F(\theta)|^2 \tag{9}$$

For $f(0)$ and $\bar{f}(0)$ we take the theoretical results of Baldini and Michetti [10]. Their values are in agreement with Ref. [11] that gives the difference $f(0)-\bar{f}(0)$ only.

We then describe $|F(\theta)|^2$ by a Gauss function,

$$|F(\theta)|^2 \approx \exp\left[-\frac{1}{2}\left(\frac{\theta}{\sigma_f}\right)^2\right]. \tag{10}$$

For the half-width σ_f we roughly expect an inverse proportionality to the momentum. Experimental data exist for $\rm K^\pm$ scattering on carbon nuclei at 800 MeV/c [9] and indicate $\sigma_f \approx 6.8°$. For lower momenta, we use calculations of elastic $\rm K_L$ scattering [8] to estimate σ_f. At 500 MeV/c we find a value of $\approx 9.8°$. For $\pi^+\pi^-$ decays, our results depend only weakly on the exact angular behaviour of the scattering amplitudes because the integral I_θ is dominated by the detector acceptance.

The angular acceptance of the CPLEAR detector, i.e. the probability for a scattered $\rm K^0(\overline{K}^0)$ to survive all selection criteria including the constraint fit for $\pi^+\pi^-$ decays (9C-fit) has first

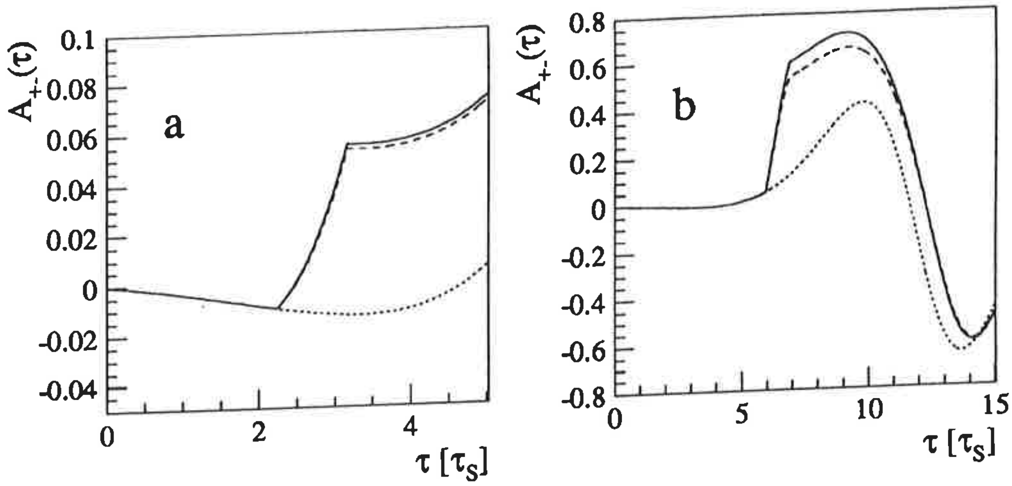

Figure 1: The $\pi^+\pi^-$ decay rate asymmetry including the effect of coherent regeneration only (full line) and with both coherent and incoherent regeneration (dashed line) for a carbon regenerator of 2.5 cm situated at a distance of 6 cm (a) and 16 cm (b) from the K^0 production point. For illustration the vacuum asymmetry is sketched in as a dotted line. (Note the different scales of the two plots.)

been studied by C. Yeche [12], and more recently and more thoroughly by Ph. Bloch [13]. The computed acceptance curves are fairly well approximated by the sum of two Gaussians,

$$A(\theta) \approx \alpha \exp\left[-\frac{1}{2}\left(\frac{\theta}{\sigma_1}\right)^2\right] + (1-\alpha)\exp\left[-\frac{1}{2}\left(\frac{\theta}{\sigma_2}\right)^2\right]. \tag{11}$$

where the values of the parameters α, σ_1 and σ_2 depend on the radius at which the scattering occurs. On the average, we find $\alpha \approx 0.75$, $\sigma_1 \approx 1.8°$ and $\sigma_2 \approx 4°$.

With (10) and (11) we arrive at the following rough estimate for the θ integration (9):

$$I_\theta \approx 2 \times 10^{-3} \tag{12}$$

Having made these simplifications it is a straightforward (albeit time consuming) task to compute the number of additional $\pi^+\pi^-$ decays caused by incoherent regeneration for any geometry as long as we can compute the time the particle spends inside the regenerator.

5 The effect of incoherent regeneration on A_{+-}

To illustrate the effect of incoherent regeneration on the $\pi^+\pi^-$ decay rate asymmetry (1) we plot $A_{+-}(\tau)$ including the additional decays from incoherent regeneration for two situations. Figure 1a shows A_{+-} for a carbon regenerator of 2.5 cm thickness at a distance of 6 cm from the K^0 production point, corresponding roughly to the set-up of the CPLEAR regeneration measurement. In Fig. 1b, the same regenerator is at a distance of 16 cm. Both curves were calculated for mono-energetic neutral kaons with 500 MeV/c momentum and with I_θ equal to the estimate (12).

It can be seen that the effect of incoherent regeneration is small compared to coherent regeneration as long as the material is situated within a few K_S life times of the K^0 production point. Beyond this region, incoherent regeneration starts to play an important role, eventually becoming the dominant effect.

6 The impact on the regeneration measurement

When we repeat the analysis of the regeneration data [14] taken in 1996 with the inclusion of $\pi^+\pi^-$-decays from incoherently regenerated K_S, using our estimate (12) for I_θ, we find changes that are smaller than the resolution of our plots, i.e. the effect is completely negligible in that case. This is expected because the position of the carbon regenerator is well inside the region of interference between inherent and coherently regenerated K_S where coherent regeneration is by far the dominant effect.

7 The impact on the $\pi^+\pi^-$ CP violation measurement

The investigation of the effect of incoherent regeneration on our results for the CP violation parameters in $\pi^+\pi^-$ decays is done by running the analysis with a regeneration correction code on the data of run periods P28 and P29 (1995) in the handy nano-DST format. It turns out that the effect is not quite negligible and we need a better estimate for the θ integration than (12).

Most of the scattering responsible for incoherent regeneration effects occurs in the proportional chambers PC1 and PC2 as well as in the rohacell wall situated between the proportional and the drift chambers (DC shell). By repeating the analysis for various regeneration corrections involving different combinations of these three walls, we find that the DC shell roughly accounts for 60% of the effect on ϕ_{+-} whereas the two proportional chambers each contribute about 20%. We therefore use an 1:1:3 average of the acceptance curves described in Ref. [13] to perform the θ integration I_θ (see Table 1).

To further increase the precision of our estimate, we evaluate I_θ for three different values of σ_f, the width of the scattering amplitudes, corresponding to three momentum ranges for the neutral kaon. Based on [8] and [9], we use $\sigma_f = 15°$ for neutral kaon momenta below 400 MeV/c, $\sigma_f = 10°$ up to 600 MeV/c and $\sigma_f = 7.5°$ above. This results in the values for I_θ given in Table 1.

Table 1: Values of the integral I_θ (Eq. 9) for various scattering radii and widths σ_f

scatterer	α	σ_1	σ_2	$I_\theta(7.5°)$	$I_\theta(10°)$	$I_\theta(15°)$
		°	°	10^{-3}	10^{-3}	10^{-3}
PC1	0.78	1.65	3.47	1.30	1.37	1.42
PC2	0.76	1.96	4.14	1.80	1.94	2.05
DC shell	0.74	1.78	4.77	1.98	2.18	2.36
1:1:3 average:				1.81	1.97	2.11

Using the average numbers as given in the last row of Table 1, we find the following differences between the fit results for the phase and the modulus of η_{+-} with the correction for incoherent regeneration and without this correction (*coherent* regeneration is corrected for in both cases):

$$\Delta\phi_{+-} = -0.11°$$
$$\text{abs}(\Delta|\eta_{+-}|) < 5 \times 10^{-6}$$

For the phase, the effect is almost half the size of the systmatic error due to the regeneration correction (which is the dominant systematic error) and must be taken into account in the final analysis. For the modulus, the effect is an order of magnitude smaller than our statistical and systematic errors and can therefore be neglected.

From the spread of the numbers in Table 1 we can deduce a rough estimate of the error on our correction. The uncertainties in σ_f and the detector acceptance propagate to changes in I_θ that do not exceed 10-20% of its value. I_θ enters the correction roughly linear (for not too large deviations) so that our correction should be accurate to better than ±20%.

8 The impact on measurements with other decay channels

8.1 $\pi^0\pi^0$ final state

The first three sections of this note apply to both charged and neutral two-pion decays. When it comes to describing the angular detector acceptance, however, things look quite different for the decay into neutral pions. In the analysis of neutral final states there is no constraint fit that effectively rejects events in which the neutral kaon was subject to scattering at a finite angle. The integral (9) therefore reduces to

$$I_\theta = \int d\theta\, \theta |F(\theta)|^2 \qquad (13)$$

and is completely determined by σ_f, the width of $|F(\theta)|^2$. Due to this strong dependence of the correction on σ_f we further subdivide the neutral kaon momentum spectrum and use the additional intermediate values $\sigma_f = 8.5°$ for momenta around 600 MeV/c and $\sigma_f = 12.5°$ for momenta around 400 MeV/c where only few events are present in the $\pi^0\pi^0$ data sample. We then get values of I_θ between 0.017 and 0.05, i.e., about an order of magnitude larger than in the case of $\pi^+\pi^-$ decays. Finally, we find the following changes in the CP violation parameters due to incoherent regeneration:

$$\Delta\phi_{00} = -0.33°$$
$$\Delta|\eta_{00}| = -3 \times 10^{-5}$$

The uncertainty of about one degree in σ_f translates to $\pm 0.16°$ in ϕ_{00} and $\pm 3 \times 10^{-5}$ in η_{00}.

8.2 3π final state

The K_L-amplitude is hardly affected by regeneration, whether coherent or incoherent, because of its almost constant (large) size over the entire range of lifetimes measured. Regeneration effects of any kind are therefore tiny in the analysis of neutral kaons decaying to three pions [15].

8.3 Semileptonic final state

More care must be taken when looking at semileptonic decays. With these decays, we directly probe the strangeness of the neutral kaon which is changed by regeneration. However, in both the T- and the CPT-violation analysis, the full change in the asymmetries associated with the correction due to coherent regeneration (not the error) is already smaller than half of the statistical uncertainty. Hence no investigation of incoherent effects is needed.

For our Δm measurement, it is shown in Ref. [5] that regeneration effects cancel in first order.

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