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# CONTRIBUTION OF CPLEAR TO THE PHYSICS OF THE NEUTRAL KAON SYSTEM

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for the CPLEAR collaboration

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**Abstract**

We present the physics results of the CP– and CPT–violation measurements performed by CPLEAR. CPLEAR has experimentally determined, for the first time, the violation of T invariance and is able to disentangle all the CP– and CPT–violating quantities from each other. This allows each of the CPT violating parameters to be determined with a precision of a few $10^{-4}$ and, in particular, the mass and width equality between the $\mathrm{K}^0$and $\overline{\mathrm{K}}^0$to be tested down to the level of $10^{-19}$ GeV. Moreover, the precision of the CPLEAR measurements allows us to probe for the first time physics on a scale approaching the Planck mass.

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# 1 Introduction

According to our present knowledge of weak interactions, the three discrete symmetries C, P and T are not exact symmetries in our universe, and the same applies to the combined symmetries CP, CT and PT. This fact has been well established experimentally and well accommodated in the Standard Model, but its origin is not yet fully understood. However, within the framework of a local field theory, of Lorentz invariance and of the usual spin-statistics requirement, any order of the triple product CPT of these discrete symmetries represents an exact symmetry expressed by the CPT theorem[1].

For the experimental verification of the validity of CPT invariance, the following points merit consideration:

- The most commonly used CPT tests, such as lifetime and mass equalities between particles and antiparticles, do not verify the full extent of CPT invariance [2].
- The set of measurements of all related variables is incomplete; therefore, their claimed experimental accuracy depends very often on hidden theoretical assumptions [3],[4] or experimental intercorrelations between measured quantities[5].
- Another fundamental aspect is the relation of CPT symmetry to the existence of the 'time arrow' through causality and gravity. Quantum gravity suggests, according to Hawking[6], that quantum field theory should be modified in such a way that pure quantum–mechanical states evolve into mixed states, which would necessarily entail a violation of CPT[7]. Such a modification in the neutral-kaon system induces three new parameters (denoted by $\alpha$, $\beta$ and $\gamma$ in Ref. [8]), describing the loss of quantum coherence in the observed system [9].

At present, the neutral-kaon system remains the most precise laboratory for measuring the totality of the parameters which describe, in the most general way, the exactness of discrete symmetries. CPLEAR has successfully developed a new experimental approach[10] to measuring the relevant parameters in the main decay modes. The method is based on the measurement of time-dependent decay-rate asymmetries in the range 0 to 20 $\tau_S$ between particles ($\mathrm{K}^0$) and antiparticles ($\overline{\mathrm{K}}^0$) by tagging the strangeness of the neutral kaons at the time of production and of decay. This possibility of defining the strangeness of a neutral kaon at both the production and the decay, unique to the CPLEAR experiment, is a superior tool for making unbiased detailed tests of discrete symmetries and CPT in particular.

The current status of the CPLEAR measurements — several reported at this workshop [11] — is shown in Table 1. In this work, we discuss the physics to be derived from the CPLEAR measurements. In Section 2, we recall the relevant parameters which describe the neutral-kaon system. In Section 3, we review the measurements and their interpretation with respect to the semileptonic decays. The measurements of all main neutral-kaon decay channels allow a CPT test based on the Bell–Steinberger relation, as presented in Section 4. In Section 5, we determine the CPT violation parameters in the $2\pi$ decays and discuss, in Section 6, the neutral-kaon mass and width equality.

Table 1: The current status of the CPLEAR measurements with detailed and total errors

| | | | |
|---|---|---|---|
| $\tau_S$ | | analysis not completed | |
| $\Delta m$ | $(529.2 \pm 1.8_{stat} \pm 0.5_{syst})10^7\hbar s^{-1}$ | $(529.2 \pm 1.9)10^7\hbar s^{-1}$ | |
| $\|\eta_{+-}\|$ | $(2.316 \pm 0.025_{stat} \pm 0.030_{syst})10^{-3}$ | $(2.316 \pm 0.039)10^{-3}$ | |
| $\varphi_{+-}$ | $(43.5 \pm 0.5_{stat} \pm 0.5_{syst} \pm 0.4_{\Delta m})^o$ | $(43.5 \pm 0.8)^o$ | |
| $\|\eta_{00}\|$ | $(2.49 \pm 0.40_{stat} \pm 0.23_{syst})10^{-3}$ | $(2.49 \pm 0.46)10^{-3}$ | 50% of statistics |
| $\varphi_{00}$ | $(51.7 \pm 7.1_{stat} \pm 1.5_{syst} \pm 0.4_{\Delta m})^o$ | $(51.7 \pm 7.3)^o$ | 50% of statistics |
| $Re\eta_{+-0}$ | $(-4 \pm 8_{stat} \pm 2_{syst})10^{-3}$ | $(-4 \pm 8)10^{-3}$ | |
| $Im\eta_{+-0}$ | $(-3 \pm 10_{stat} \pm 2_{syst})10^{-3}$ | $(-3 \pm 10)10^{-3}$ | |
| $Re\eta_{000}$ | $0.15 \pm 0.30_{stat} \pm 0.04_{syst}$ | $0.15 \pm 0.30$ | 25% of statistics |
| $Im\eta_{000}$ | $0.29 \pm 0.40_{stat} \pm 0.03_{syst}$ | $0.29 \pm 0.40$ | 25% of statistics |
| $\mathrm{A_T}$ | $(6.3 \pm 2.1_{stat} \pm 1.8_{syst})10^{-3}$ | $(6.3 \pm 2.8)10^{-3}$ | 30% of statistics |
| $\mathrm{A_{CPT}}$ | $(0.28 \pm 2.1_{stat} \pm 1.8_{syst})10^{-3}$ | $(0.28 \pm 2.8)10^{-3}$ | 30% of statistics |
| $Rex$ | $(8.5 \pm 7.5_{stat} \pm 6.9_{syst})10^{-3}$ | $(8.5 \pm 10.2)10^{-3}$ | 30% of statistics |
| $Im(x+\delta)$ | $(0.5 \pm 2.4_{stat} \pm 0.6_{syst})10^{-3}$ | $(0.5 \pm 2.5)10^{-3}$ | 30% of statistics |
| $\alpha$ | | $(-0.5 \pm 2.8)10^{-17}$ GeV | 30% of statistics |
| $\beta$ | | $(2.5 \pm 2.3)10^{-19}$ GeV | 30% of statistics |
| $\gamma$ | | $(1.1 \pm 2.5)10^{-21}$ GeV | 30% of statistics |

## 2 The kaon phenomenology

The parameters of T and CPT invariance in the kaon system can be found in many textbooks and reviews (see f.i. [2], [12], [13], [14]), and only a brief outline will be presented here for clarity and completeness. The two neutral-kaon eigenstates of weak interaction $\binom{\mathrm{K_S}}{\mathrm{K_L}}$ are given from the strangeness eigenstates $\binom{\mathrm{K^0}}{\overline{\mathrm{K}}^0}$ by

$$\begin{pmatrix}\mathrm{K_S}\\ \mathrm{K_L}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1+\varepsilon_S}{\sqrt{|\varepsilon_S|^2+1}} & \frac{1-\varepsilon_S}{\sqrt{|\varepsilon_S|^2+1}}\\ \frac{1+\varepsilon_L}{\sqrt{|\varepsilon_L|^2+1}} & -\frac{1-\varepsilon_L}{\sqrt{|\varepsilon_L|^2+1}}\end{pmatrix}\begin{pmatrix}\mathrm{K^0}\\ \overline{\mathrm{K}}^0\end{pmatrix} \tag{1}$$

and their time evolution is

$$\begin{pmatrix}\Psi_{\mathrm{K_S}}(t)\\ \Psi_{\mathrm{K_L}}(t)\end{pmatrix} = \begin{pmatrix}\mathrm{a}_S\mathrm{K_S}\\ \mathrm{a}_L\mathrm{K_L}\end{pmatrix} \tag{2}$$

where $\mathrm{a}_{S,L} = \exp(-i\mathrm{M}_{S,L}t)$, $\mathrm{M}_S = m_S - \frac{i}{2}\Gamma_S$, $\mathrm{M}_L = m_L - \frac{i}{2}\Gamma_L$ and $t$ is the proper time.

An initially pure strangeness state is given by

$$\Psi(t) \equiv \begin{pmatrix}\Psi_{\mathrm{K^0}}(t)\\ \Psi_{\overline{\mathrm{K}}^0}(t)\end{pmatrix} = \frac{1}{2}\frac{1}{1-\varepsilon_L\varepsilon_S}\left( \begin{bmatrix}(\varepsilon_S-\varepsilon_L)+(1-\varepsilon_S\varepsilon_L) & -(\varepsilon_S+\varepsilon_L)+(1+\varepsilon_S\varepsilon_L)\\ (\varepsilon_S+\varepsilon_L)+(1+\varepsilon_S\varepsilon_L) & -(\varepsilon_S-\varepsilon_L)+(1-\varepsilon_S\varepsilon_L)\end{bmatrix}\mathrm{a}_S \right.$$
$$\left. + \begin{bmatrix}-(\varepsilon_S-\varepsilon_L)+(1-\varepsilon_S\varepsilon_L) & (\varepsilon_S+\varepsilon_L)-(1+\varepsilon_S\varepsilon_L)\\ -(\varepsilon_S+\varepsilon_L)-(1+\varepsilon_S\varepsilon_L) & (\varepsilon_S-\varepsilon_L)+(1-\varepsilon_S\varepsilon_L)\end{bmatrix}\mathrm{a}_L \right)\begin{pmatrix}\mathrm{K^0}\\ \overline{\mathrm{K}}^0\end{pmatrix} \tag{3}$$

or, with $\varepsilon_S = \varepsilon + \delta$ and $\varepsilon_L = \varepsilon - \delta$ where $\varepsilon$ is the T–violation/CPT–invariance parameter and

$\delta$ the T-invariance/CPT-violation parameter, and neglecting terms of the order of $(\varepsilon_S\varepsilon_L)$,

$$\Psi(t) = \frac{1}{2}\left(\begin{bmatrix} 2\delta{+}1 & -2\varepsilon{+}1 \\ 2\varepsilon{+}1 & -2\delta{+}1 \end{bmatrix} \mathrm{a}_S + \begin{bmatrix} -2\delta{+}1 & 2\varepsilon{-}1 \\ -2\varepsilon{-}1 & 2\delta{+}1 \end{bmatrix} \mathrm{a}_L\right) \begin{pmatrix} \mathrm{K}^0 \\ \overline{\mathrm{K}^0} \end{pmatrix} . \tag{4}$$

Here the non-vanishing parameters $\varepsilon$ and $\delta$ are chosen so as to describe, in the $\Delta S = 2$ transitions, the non-conservation of T and CPT symmetry, respectively. The Wigner–Weisskopf evolution of this state is given by

$$\frac{d}{dt}\Psi = -i\Lambda\Psi \ , \quad \text{where} \quad \Lambda \equiv \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix} = \begin{pmatrix} \mathrm{M}_{11} & \mathrm{M}_{12} \\ \mathrm{M}_{21} & \mathrm{M}_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} , \tag{5}$$

and where M and $\Gamma$ are Hermitian matrices. From the two independent matrix equations for $\mathrm{K_S}$ and $\mathrm{K_L}$ we obtain

$$\varepsilon = \frac{\Lambda_{12} - \Lambda_{21}}{2(\mathrm{M}_L - \mathrm{M}_S)} = \left(Im\mathrm{M}_{12} - \frac{i}{2}Im\Gamma_{12}\right)\frac{i\Delta m + \Delta\gamma/2}{\Delta m^2 + (\Delta\gamma/2)^2} \tag{6}$$

where $\Delta m = m_L - m_S$, $\Delta\gamma = \Gamma_S - \Gamma_L$ and

$$\delta = \frac{\Lambda_{22} - \Lambda_{11}}{2(\mathrm{M}_L - \mathrm{M}_S)} = -\frac{i}{2}\left[(\mathrm{M}_{22} - \mathrm{M}_{11}) - \frac{i}{2}(\Gamma_{22} - \Gamma_{11})\right]\frac{i\Delta m + \Delta\gamma/2}{\Delta m^2 + (\Delta\gamma/2)^2} . \tag{7}$$

With the convention $Im\Gamma_{12} = 0$, we have:

$$\varepsilon = \frac{Im\mathrm{M}_{12}}{\sqrt{\Delta m^2 + (\Delta\gamma/2)^2}}\exp(i\varphi_{SW}) = |\varepsilon|\exp(i\varphi_{SW}) \tag{8}$$

where $\varphi_{SW} = \arctan(2\Delta m/\Delta\gamma)$, and

$$\begin{aligned} \delta &= \frac{1}{2}\sqrt{\frac{(\mathrm{M}_{22} - \mathrm{M}_{11})^2 + \left(\frac{1}{2}(\Gamma_{22} - \Gamma_{11})\right)^2}{\Delta m^2 + (\Delta\gamma/2)^2}}\exp\left(i(\varphi_{SW} - \varphi_{\mathrm{CPT}} - \frac{\pi}{2})\right) \\ &= |\delta|\exp\left(i(\varphi_{SW} - \varphi_{\mathrm{CPT}} - \frac{\pi}{2})\right) \end{aligned} \tag{9}$$

with $\varphi_{\mathrm{CPT}} = \arctan(\frac{1}{2}(\Gamma_{22} - \Gamma_{11})/(\mathrm{M}_{22} - \mathrm{M}_{11}))$

Assuming CPT invariance in the strong interactions, one can introduce for the $\Delta S = 1$ current the amplitudes $\mathrm{A_I}$ and $\mathrm{B_I}$ [15]. These are the CPT-conserving and CPT-violating amplitudes of isospin I in the decay of the neutral kaon. We assume $Im\mathrm{B_I}$ to be negligible, because they contradict the unitarity of the S-matrix, the hermiticity of the Hamiltonian [15]. For the two-pion final states we define two CP-violation parameters

$$\eta_{+-} \equiv \frac{\mathrm{A}_L^{+-}}{\mathrm{A}_S^{+-}} \qquad \eta_{00} \equiv \frac{\mathrm{A}_L^{00}}{\mathrm{A}_S^{00}} \tag{10}$$

for the $\pi^+\pi^-$ and $\pi^0\pi^0$ decays respectively. We obtain to first order in CP, T and CPT-violation parameters

$$\eta_{+-} = \varepsilon - \delta + \mathrm{a} + (\varepsilon' - \mathrm{a}\omega) \tag{11}$$

$$\eta_{00} = \varepsilon - \delta + \mathrm{a} - 2(\varepsilon' - \mathrm{a}\omega) \tag{12}$$

where

$$\mathrm{a} \equiv \frac{<\mathrm{I}=0|\mathrm{H}_w|\mathrm{K}_2>}{<\mathrm{I}=0|\mathrm{H}_w|\mathrm{K}_1>} = \frac{Re\mathrm{B}_0 + iIm\mathrm{A}_0}{Re\mathrm{A}_0 + iIm\mathrm{B}_0} \approx \frac{Re\mathrm{B}_0}{Re\mathrm{A}_0} + i\frac{Im\mathrm{A}_0}{Re\mathrm{A}_0} \ , \tag{13}$$

$$\begin{aligned} \varepsilon' &\equiv \frac{1}{\sqrt{2}}\frac{<\mathrm{I}=2|\mathrm{H}_w|\mathrm{K}_2>}{<\mathrm{I}=0|\mathrm{H}_w|\mathrm{K}_1>} = \frac{1}{\sqrt{2}}e^{i(\delta_2-\delta_0)}\frac{Re\mathrm{B}_2 + iIm\mathrm{A}_2}{Re\mathrm{A}_0 + iIm\mathrm{B}_0} \\ &\approx \frac{1}{\sqrt{2}}\left[\frac{Re\mathrm{B}_2}{Re\mathrm{A}_0}e^{i(\delta_2-\delta_0)} + \frac{Im\mathrm{A}_2}{Re\mathrm{A}_0}e^{i(\delta_2-\delta_0+\frac{\pi}{2})}\right] = \varepsilon'_{\perp} + \varepsilon'_{\parallel} \ , \end{aligned} \tag{14}$$

$$\omega \equiv \frac{1}{\sqrt{2}}\frac{<\mathrm{I}=2|\mathrm{H}_w|\mathrm{K}_1>}{<\mathrm{I}=0|\mathrm{H}_w|\mathrm{K}_1>} = \frac{1}{\sqrt{2}}e^{i(\delta_2-\delta_0)}\frac{Re\mathrm{A}_2 + iIm\mathrm{B}_2}{Re\mathrm{A}_0 + iIm\mathrm{B}_0} \approx \frac{1}{\sqrt{2}}\frac{Re\mathrm{A}_2}{Re\mathrm{A}_0}e^{i(\delta_2-\delta_0)} \ . \tag{15}$$

Here $|\mathrm{K}_1>$ and $|\mathrm{K}_2>$ are the CP = +1 and CP = –1 states, respectively, and $\delta_I$ are the strong interaction phase shifts for the isospin I amplitudes.

Similarly, for the three-pion final states we define the corresponding CP-violation parameters as

$$\eta_{+-0} \equiv \frac{\mathrm{A}_S^{+-0}}{\mathrm{A}_L^{+-0}} \qquad \eta_{000} \equiv \frac{\mathrm{A}_S^{000}}{\mathrm{A}_L^{000}} \ . \tag{16}$$

For the semileptonic decays, there are four amplitudes:

$$\begin{aligned} \mathrm{A}^+ &= \mathrm{A}\Big(\mathrm{K}^0 \to \pi^-\ell^+\nu\Big) = \mathrm{A}_\ell + \mathrm{B}_\ell & (17)\\ \mathrm{A}^- &= \mathrm{A}\Big(\mathrm{K}^0 \to \pi^+\ell^-\bar{\nu}\Big) = x^*\mathrm{A}_\ell^* + \xi^*\mathrm{B}_\ell^* & (18)\\ \bar{\mathrm{A}}^+ &= \mathrm{A}\Big(\overline{\mathrm{K}}^0 \to \pi^-\ell^+\nu\Big) = x\ \mathrm{A}_\ell - \xi\mathrm{B}_\ell & (19)\\ \bar{\mathrm{A}}^- &= \mathrm{A}\Big(\overline{\mathrm{K}}^0 \to \pi^+\ell^-\bar{\nu}\Big) = \mathrm{A}_\ell^* - \mathrm{B}_\ell^* & (20) \end{aligned}$$

where $\mathrm{A}_\ell$ ($\mathrm{B}_\ell$) is the CPT-conserving (CPT-violating) decay amplitude and the CPT-violation in semileptonic decays is measured by the parameter $y = \mathrm{B}_\ell/\mathrm{A}_\ell$. The parameter $x$ ($\xi$) measures the violation of the $\Delta S = \Delta Q$ rule in the CPT-conserving (CPT-violating) decays. The parameter $x$ in the Standard Model is expected to be extremely small, at most $10^{-6}$ [16].

# 3 Direct measurement of T violation and CPT invariance in the $\Delta S = 2$ transitions

CPLEAR can provide a direct evidence of T violation as outlined below. Since weak interactions do not conserve strangeness, a $\mathrm{K}^0$ transforms itself into a $\overline{\mathrm{K}}^0$ in the course of time (Eq. 3). Similarly, a state produced initially as $\overline{\mathrm{K}}^0$ may be found later to have changed into $\mathrm{K}^0$. Time reversal invariance, or microscopic reversibility, would require all details of the second process to be deducible from the first [17]; in particular, $\overline{\mathrm{K}}^0 \to \mathrm{K}^0$ should proceed at a rate exactly equal to that of $\mathrm{K}^0 \to \overline{\mathrm{K}}^0$, since time reversal simply reverses the process. We have

$$
\begin{aligned}
&\langle \mathrm{K}^0|\mathrm{M}-\frac{i}{2}\Gamma|\overline{\mathrm{K}}^0\rangle = \langle \mathrm{TK}^0|\mathrm{T}(\mathrm{M}-\frac{i}{2}\Gamma)|\overline{\mathrm{K}}^0\rangle^* = \langle \mathrm{TK}^0|\mathrm{T}(\mathrm{M}-\frac{i}{2}\Gamma)\mathrm{T}^{-1}|\mathrm{T}\overline{\mathrm{K}}^0\rangle^* = \\
&\langle \mathrm{TK}^0|(\mathrm{TMT}^{-1})+\frac{i}{2}(\mathrm{T}\Gamma\mathrm{T}^{-1})|\mathrm{T}\overline{\mathrm{K}}^0\rangle^* = \langle \mathrm{T}\overline{\mathrm{K}}^0|(\mathrm{TMT}^{-1})^\dagger+(\frac{i}{2}\mathrm{T}\Gamma\mathrm{T}^{-1})^\dagger|\mathrm{TK}^0\rangle = \\
&\langle \mathrm{T}\overline{\mathrm{K}}^0|(\mathrm{M}_T)^\dagger+(\frac{i}{2}\Gamma_T)^\dagger|\mathrm{TK}^0\rangle = \langle \mathrm{T}\overline{\mathrm{K}}^0|\mathrm{M}_T-\frac{i}{2}\Gamma_T|\mathrm{TK}^0\rangle = \langle \overline{\mathrm{K}}^0|\mathrm{M}_T-\frac{i}{2}\Gamma_T|\mathrm{K}^0\rangle
\end{aligned}
\tag{21}
$$

with $\mathrm{M}_T(\Gamma_T) = \mathrm{TM}(\Gamma)\mathrm{T}^{-1}$ and $|\mathrm{K}^0\rangle = \mathrm{T}|\mathrm{K}^0\rangle$, $|\overline{\mathrm{K}}^0\rangle = \mathrm{T}|\overline{\mathrm{K}}^0\rangle$ within a phase angle. Therefore, the rates are equal if the underlying theory is invariant under T transformation, that is if $\mathrm{M}_T(\Gamma_T) = \mathrm{M}(\Gamma)$, without any additional assumption on CPT invariance.

A convenient way to make the measurement is to take advantage of the $\Delta S = \Delta Q$ rule. Then $\pi^-\ell^+\nu$ and $\pi^+\ell^-\bar{\nu}$ decays in a neutral-kaon beam provide indicators of the $\mathrm{K}^0$ and $\overline{\mathrm{K}}^0$ content, respectively. The first could be used in decays of a beam produced initially as $\overline{\mathrm{K}}^0$ to measure $\overline{\mathrm{K}}^0 \to \mathrm{K}^0$ conversion, while the second would similarly yield the rate of $\mathrm{K}^0 \to \overline{\mathrm{K}}^0$ transitions. Since CPLEAR has the unique feature of tagging the strangeness of the neutral kaon both at its production and at its decay, it is able to compare the probability $|\langle \mathrm{K}^0|\mathrm{M}-\frac{i}{2}\Gamma|\overline{\mathrm{K}}^0\rangle|^2$ with the probability $|\langle \overline{\mathrm{K}}^0|\mathrm{M}-\frac{i}{2}\Gamma|\mathrm{K}^0\rangle|^2$ by forming the asymmetry

$$
\mathrm{A_T}(t) \equiv \frac{|\bar{\mathrm{A}}^+|^2-|\mathrm{A}^-|^2}{|\bar{\mathrm{A}}^+|^2+|\mathrm{A}^-|^2} \quad (= 4Re\varepsilon + 2Rey \quad \text{at long lifetimes}). \tag{22}
$$

A fit to only one third of the total CPLEAR data gives

$$
\mathrm{A_T} = (\mathbf{6.3 \pm 2.1_{stat} \pm 1.8_{syst}}) \times \mathbf{10^{-3}} \tag{23}
$$

which represents the first direct experimental evidence of T non-invariance.

Similarly, CPT invariance requires that the fraction of $\mathrm{K}^0$s which survive as $\mathrm{K}^0$ at time $t$ is exactly equal to that of $\overline{\mathrm{K}}^0$s which remain as $\overline{\mathrm{K}}^0$. Given its tagging capacity, CPLEAR measures at the proper time $t$ the fraction of $\mathrm{K}^0$s arising from pure $\mathrm{K}^0$ states and the fraction of $\overline{\mathrm{K}}^0$s arising from pure $\overline{\mathrm{K}}^0$ states. From these rates the following asymmetry is formed:

$$
\mathrm{A_{CPT}}(t) \equiv \frac{|\bar{\mathrm{A}}^-|^2-|\mathrm{A}^+|^2}{|\bar{\mathrm{A}}^-|^2+|\mathrm{A}^+|^2} \quad (= 4Re\delta - 2Rey \quad \text{at long lifetimes}). \tag{24}
$$

A fit to one third of the available statistics gives a preliminary value for $Re\delta$:

$$
Re\delta = (\mathbf{0.07 \pm 0.53_{stat} \pm 0.45_{syst}}) \times \mathbf{10^{-3}} \quad \text{for } Rey = 0\ . \tag{25}
$$

For short lifetimes, the asymmetry $\mathrm{A_{CPT}}$ is also sensitive to $Im\delta$, but the analysis is not yet complete.

Moreover, CPLEAR determines from Eqs. 22 and 24, free from the assumption of CPT invariance in the decays,

$$
Re\varepsilon_S = \frac{1}{4}(\mathrm{A_T}+\mathrm{A_{CPT}}) = (\mathbf{1.6 \pm 0.7_{stat} \pm 0.6_{syst}}) \times \mathbf{10^{-3}} \tag{26}
$$

where only one third of the CPLEAR statistics is analysed.

As noted by Michel[18], usually a T violation effect in the neutral-kaon system cannot be directly separated from CPT violation. The advantage of CPLEAR is that of addressing separately $\varepsilon$ and $\delta$, thus the measured effects are much more decisive.

Finally, through a common fit of the four rates $\mathrm{R}\Big[\mathrm{K}^0 \to \pi^-\ell^+\nu\Big](t)$, $\mathrm{R}\Big[\mathrm{K}^0 \to \pi^+\ell^-\bar{\nu}\Big](t)$, $\mathrm{R}\Big[\overline{\mathrm{K}}^0 \to \pi^-\ell^+\nu\Big](t)$ and $\mathrm{R}\Big[\overline{\mathrm{K}}^0 \to \pi^+\ell^-\bar{\nu}\Big](t)$, CPLEAR will provide values for each of the parameters $Re\varepsilon$, $Re\delta$, $Im\delta$, $x$, $\xi$ and $y$.

# 4 Determination of $Im\delta$ from the Bell–Steinberger relation

The world's best limits of CPT invariance are obtained by comparing the phase $\varphi_{+-}$ with the superweak phase $\varphi_{SW}$. Given the different strong correlation of the measurement of $\varphi_{+-}$ and $\Delta m$ for most of the experiments, an adequate evaluation procedure has been proposed by CPLEAR [19]. Figure 1 shows the experimental data available up to 1992 together with the results from CPLEAR. The systematic errors of Ref. [5] and [20] concerning their phase measurements, mainly determined by the regeneration amplitudes calculation [21], have been the subject of discussion [21],[22],[23]. Clearly, the dependence of a test on the validity of CPT invariance on theoretical assumptions is a matter of concern [24]. Therefore, in the fit of Fig. 1 these experiments have not been included. The results of this fit (which include the full CPLEAR statistics) are

$$\varphi_{+-} = (43.7 \pm 0.8)^o \quad ; \quad \Delta m = (530.5 \pm 1.3) \times 10^7 \hbar\mathrm{s}^{-1} \quad ; \quad \varphi_{SW} = (43.49 \pm 0.08)^o \ . \tag{27}$$

Including also the results of Ref. [5] and [20] we find (see Fig. 2)

$$\varphi_{+-} = (43.6 \pm 0.6)^o \quad ; \quad \Delta m = (530.1 \pm 1.1) \times 10^7 \hbar\mathrm{s}^{-1} \quad ; \quad \varphi_{SW} = (43.46 \pm 0.08)^o \ . \tag{28}$$

In both cases, the correlated fits to the data provide a value of $\varphi_{+-}$ in good agreement with the superweak phase, the most precise value being $(\varphi_{+-} - \varphi_{SW}) = (0.1 \pm 0.6)^o$. We are happy that our method [19] is adopted in the 1996 Review of Particle Properties by the Particle Data Group [3].

These precise values can be used in the determination of $Re\varepsilon$ and $Im\delta$ through the Bell–Steinberger relation. This relation leads [14],[25] to

$$\begin{aligned} -iIm\delta + Re\varepsilon = & \frac{1}{2(i\Delta m + \frac{1}{2}\gamma)} \times \\ & \Big(\sum\Big(|\mathrm{A}_S|^2\eta_{\pi\pi}\Big) + \sum\Big(|\mathrm{A}_L|^2\eta^*_{3\pi}\Big) + 2\Big[-i(Imx + Im\delta) + Re\varepsilon + Rey\Big]|f|^2\Big) \end{aligned} \tag{29}$$

where $\gamma = \Gamma_S + \Gamma_L$, $|\mathrm{A}_S|^2 \approx BR(\mathrm{K_S} \to \pi\pi)\Gamma_S$, $|\mathrm{A}_L|^2 \approx BR(\mathrm{K_L} \to \pi\pi\pi)\Gamma_L$ and $|f|^2 \approx BR(\mathrm{K_L} \to \pi\ell\nu)\Gamma_L$.

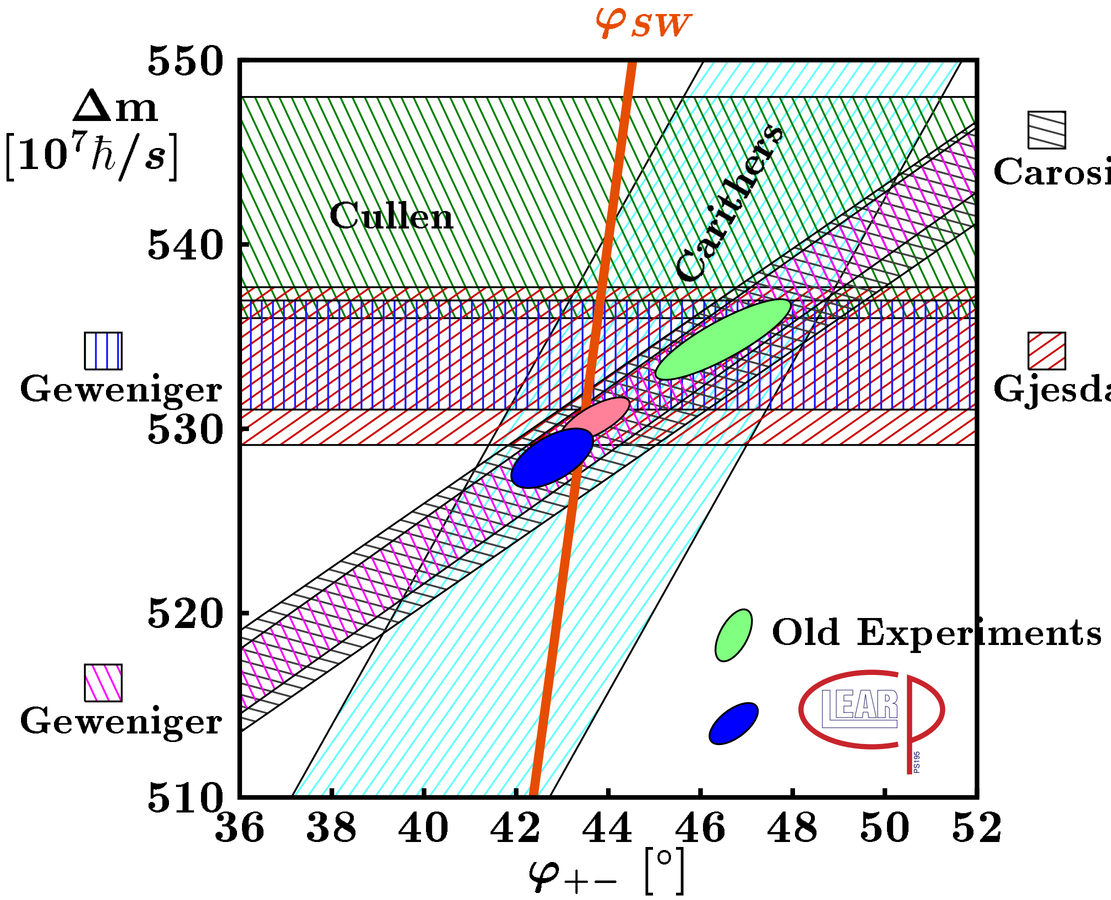


Figure 1: The plot shows the $1\sigma$ of measurements listed in Ref. [19], including the full statistics of CPLEAR. The ellipse in the center represents the result of the fit.

By using the relevant CPLEAR measurements of Table 1 and Eqs. 22, 24 and 29, together with the PDG averages [3] $r = \frac{|\eta_{00}|}{|\eta_{+-}|} = 0.9930 \pm 0.0020$ and $\varphi_{00} - \varphi_{+-} = (-0.3 \pm 0.8)^o$, we obtain by standard $\chi^2$-analysis

$$\begin{aligned} Im\delta =& \ (8.0 \pm 9.6) \times 10^{-5} \quad ; \quad Re\varepsilon = (1.63 \pm 0.09) \times 10^{-3} \\ Re\delta =& (-0.1 \pm 9.9) \times 10^{-4} \quad ; \quad Rey = (-0.1 \pm 1.4) \times 10^{-3} \\ Re\varepsilon_S =& \ (1.65 \pm 0.71) \times 10^{-3} \end{aligned} \tag{30}$$

where the error on $Im\delta$ and $Re\varepsilon$ is at present dominated by the error on $\eta_{000}$ only. The CPLEAR accuracy on $\pi^+\pi^-\pi^0$ and $\pi\ell\nu$ are such that their contributions became negligible.

The analysis of the full CPLEAR statistics of the $3\pi^0$ decays will render the contribution of $3\pi^0$ final states also negligible. Assuming provisionally $\eta_{000} = \eta_{+-0}$, we obtain

$$\begin{aligned} Im\delta =& (-0.4 \pm 1.9) \times 10^{-5} \quad ; \quad Re\varepsilon = (1.65 \pm 0.01) \times 10^{-3} \\ Re\delta =& (-0.1 \pm 9.9) \times 10^{-4} \quad ; \quad Rey = (-0.1 \pm 1.4) \times 10^{-3} \\ Re\varepsilon_S =& \ (1.65 \pm 0.71) \times 10^{-3} \end{aligned} \tag{31}$$

After completion of our semileptonic analysis and the simultaneous fit of the four semileptonic decay rates, CPLEAR will improve its precision on $Re\delta$ and $Rey$ by a factor of two.

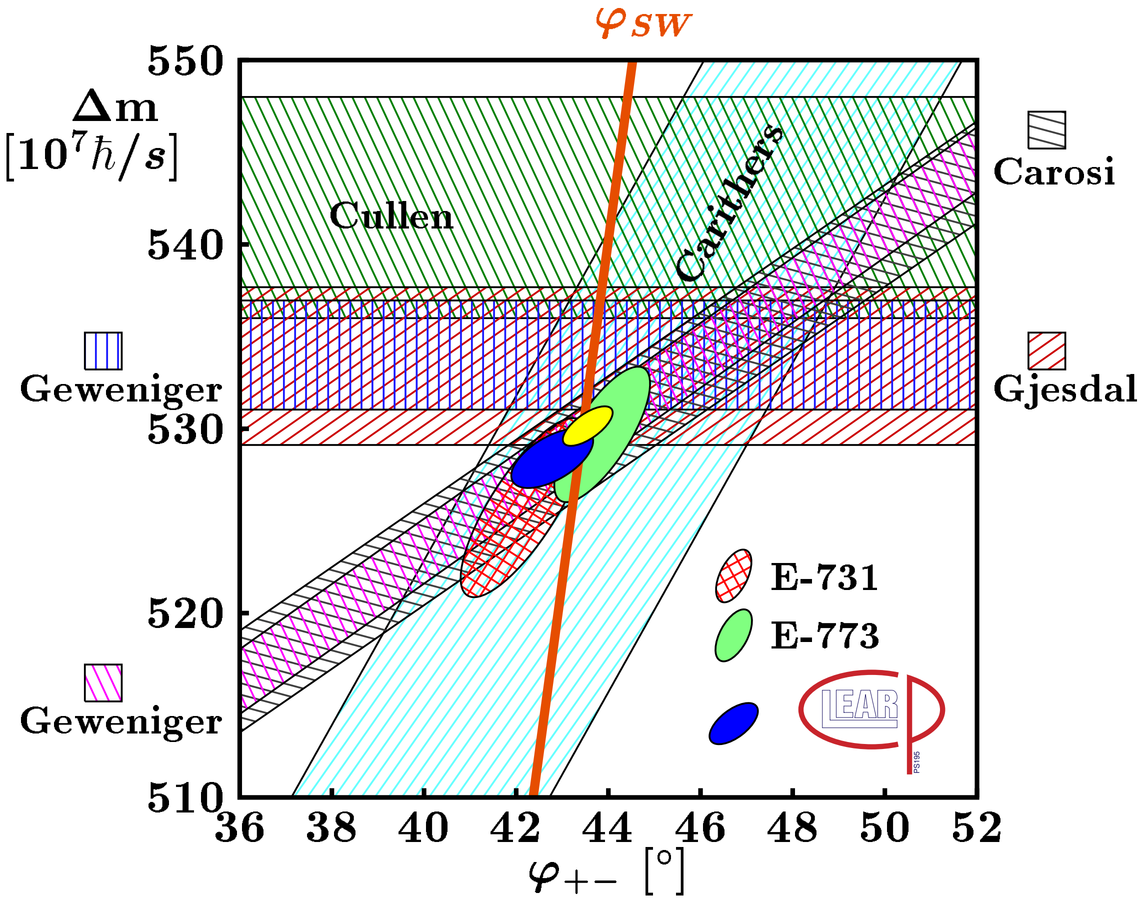


Figure 2: The plot shows the $1\sigma$ of measurements listed in Ref. [19], including the full statistics of CPLEAR. The ellipse in the center represents the result of the fit.

## 5 CP and CPT invariance in the $\Delta S = 1$ transitions

From Eq. 14 we observe that $\varepsilon'$ has two components, the first CPT-conserving ($\varepsilon'_{\parallel}$) and the second CPT-violating ($\varepsilon'_{\perp}$) perpendicular to the first. However, the quantity ($\varepsilon' - \mathrm{a}\omega$) is the most relevant[26],[13] in describing CPT-violation in the $\Delta S = 1$ transitions, because it includes the CPT-violating decays in both isospin amplitudes. From Fig. 3 it can easily be seen that

$$|\eta_{00}|\sin(\varphi_{00} - \varphi_{SW}) = |\eta_{+-}|\sin(\varphi_{+-} - \varphi_{SW}) - 3(\varepsilon' - \mathrm{a}\omega)_{\perp} \tag{32}$$

or

$$\begin{aligned}(\varepsilon' - \mathrm{a}\omega)_{\perp} &= -\frac{1}{3}|\eta_{+-}|\Big\{r\sin(\varphi_{00}{-}\varphi_{+-})\cos(\varphi_{+-}{-}\varphi_{SW}) + \sin(\varphi_{+-}{-}\varphi_{SW})\Big[r\cos(\varphi_{00}{-}\varphi_{+-}) - 1\Big]\Big\} \\ &= (0.40 \pm 1.56) \times 10^{-5}\ .\end{aligned} \tag{33}$$

Similarly, we can determine

$$\begin{aligned}(\varepsilon' - \mathrm{a}\omega)_{\parallel} &= \frac{1}{3}|\eta_{+-}|\Big\{r\sin(\varphi_{00}{-}\varphi_{+-})\sin(\varphi_{+-}{-}\varphi_{SW}) - \cos(\varphi_{+-}{-}\varphi_{SW})\Big[r\cos(\varphi_{00}{-}\varphi_{+-}) - 1\Big]\Big\} \\ &= (5.41 \pm 1.55) \times 10^{-6}\ .\end{aligned} \tag{34}$$

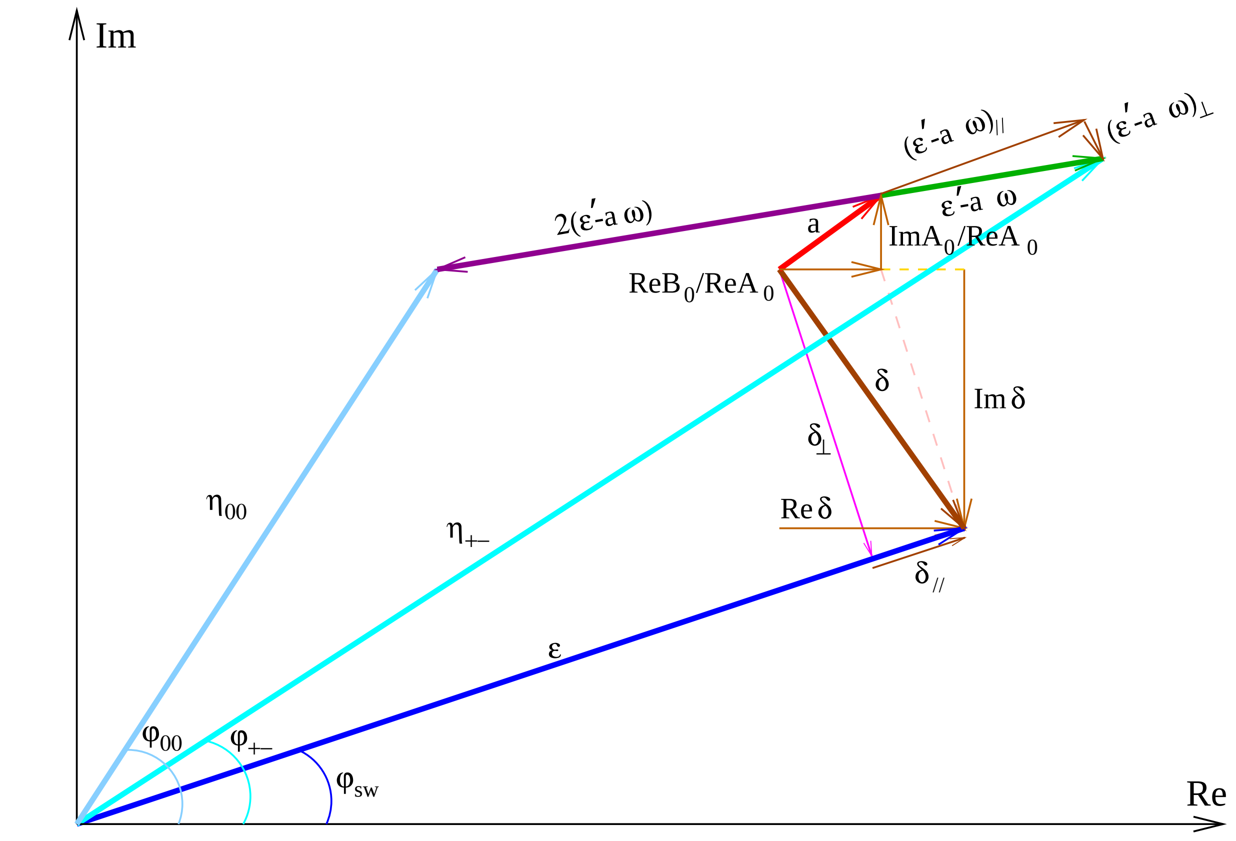


Figure 3: Schematic representation in the complex plane (Wu–Yang diagram), of the relations between $\eta_{+-}$, $\eta_{00}$, $\varepsilon$, $\delta$, a, and $(\varepsilon' - \mathrm{a}\omega)$. Relative sizes are not in scale.

Because CPLEAR has measured directly $Re\delta$ (Eq. 25) with a precision better than $10^{-3}$ and $Im\delta$ (Eq. 31), a preliminary value of $\frac{Re\mathrm{B}_0}{Re\mathrm{A}_0}$ can be determined by

$$\frac{Re\mathrm{B}_0}{Re\mathrm{A}_0} = Re\delta + Im\delta \tan\varphi_{SW} = (\mathbf{-0.14 \pm 9.90}) \times \mathbf{10^{-4}} \ . \tag{35}$$

where the CPT-violating contributions of the three-pion and semileptonic decays are neglected, since they are suppressed by $\Gamma_L/\Gamma_S$.

On the other hand, from Section 2, we can write

$$\frac{Im\mathrm{A}_0}{Re\mathrm{A}_0} = \frac{2|\eta_{+-}|}{3\cos\varphi_{SW}}\Big\{\sin(\varphi_{+-}-\varphi_{SW}) + \frac{r}{2}\sin\Big[(\varphi_{00}-\varphi_{+-}) + (\varphi_{+-}-\varphi_{SW})\Big]\Big\} + Im\delta(1+\tan^2\varphi_{SW})$$
$$= (-0.76 \pm 4.62) \times 10^{-5} \ . \tag{36}$$

Hence, we determine:

$$\frac{Re\mathrm{B}_2}{Re\mathrm{A}_0} = \sqrt{2}\Big[\frac{Re\mathrm{B}_0}{Re\mathrm{A}_0}|\omega| - \cos\Delta\big(\varepsilon' - \mathrm{a}\omega\big)_{\perp} + \sin\Delta\big(\varepsilon' - \mathrm{a}\omega\big)_{\|}\Big] = (\mathbf{-0.61 \pm 4.96}) \times \mathbf{10^{-5}} \tag{37}$$

and

$$\frac{Im\mathrm{A}_2}{Re\mathrm{A}_0} = \sqrt{2}\Big[\frac{\big(\varepsilon' - \mathrm{a}\omega\big)_{\|}}{\cos\Delta} + \frac{Im\mathrm{A}_0}{Re\mathrm{A}_0}|\omega|\Big] - \tan\Delta\Big(\frac{Re\mathrm{B}_2}{Re\mathrm{A}_0} - \sqrt{2}\frac{Re\mathrm{B}_0}{Re\mathrm{A}_0}|\omega|\Big) = (\mathbf{7.45 \pm 3.53}) \times \mathbf{10^{-6}} \tag{38}$$

where $\Delta = \delta_2 - \delta_0 + \frac{\pi}{2} - \varphi_{SW} = (1.5 \pm 6)^o$ [27] and $|\omega| = 0.03167 \pm 0.00014$ [28].

With the full-statistics results, it would be hardly possible to exclude that a direct CP-violation effect is due to CPT- rather than to only T-violation, unless $(\varphi_{+-} - \varphi_{00})$ is measured to an extraordinary accuracy (see Eq. 33). In this sense, an improved $r$ accuracy appears to be less important [26], since the CPT-violating and the CPT-conserving components of the CP-violation parameters can no longer be separated because of equally refined sensitivity.

# 6 $\mathrm{K}^0$ and $\overline{\mathrm{K}}^0$ mass and width equality

Following the formalism of Section 2, we can decompose the parameter $\delta$ (Eq. 7) into a component parallel to $\varphi_{SW}$ and another perpendicular to it:

$$\delta_{\|} = \frac{1}{4}\frac{|\Gamma_{22} - \Gamma_{11}|}{\sqrt{\Delta m^2 + (\frac{\Delta\gamma}{2})^2}} \quad \text{and} \quad \delta_{\perp} = \frac{1}{2}\frac{|\mathrm{M}_{22} - \mathrm{M}_{11}|}{\sqrt{\Delta m^2 + (\frac{\Delta\gamma}{2})^2}} \tag{39}$$

which can be expressed by the measured quantities (see Table 1)

$$\delta_{\|} = Re\delta \cos\varphi_{SW} + Im\delta \sin\varphi_{SW} \quad ; \quad \delta_{\perp} = -Re\delta \sin\varphi_{SW} + Im\delta \cos\varphi_{SW} \ . \tag{40}$$

Following Eq. 39 we determine the equality of mass and width:

$$\begin{aligned} |m_{\mathrm{K}^0} - m_{\overline{\mathrm{K}}^0}| &= (0.04 \pm 6.91) \times 10^{-18}\ \text{GeV} \quad \text{and} \\ |\Gamma_{22} - \Gamma_{11}| &= (0.02 \pm 1.46) \times 10^{-17}\ \text{GeV}\ . \end{aligned} \tag{41}$$

With the assumption of CPT invariance in the decay amplitudes,

$$|\mathbf{m}_{\mathrm{K}^0} - \mathbf{m}_{\overline{\mathrm{K}}^0}| = (\mathbf{0.56 \pm 2.65}) \times \mathbf{10^{-19}}\ \text{GeV} \quad \text{and} \quad \Gamma_{\mathbf{11}} = \Gamma_{\mathbf{22}}\ . \tag{42}$$

Therefore, CPLEAR provides the best determination on the difference in masses and widths between $\mathrm{K}^0$ and $\overline{\mathrm{K}}^0$ based exclusively on direct measurements.

# 7 Conclusions

In the past, the neutral-kaon experiments provided direct measurements of five parameters: $|\eta_{+-}|$, $\varphi_{+-}$, $|\eta_{00}|$, $\varphi_{00}$ and the $\mathrm{K_L}$ charge asymmetry. However, there are in total eight main CP-violation parameters, namely $Re\varepsilon$, $\frac{Im\mathrm{A}_0}{Re\mathrm{A}_0}$ and $\frac{Im\mathrm{A}_2}{Re\mathrm{A}_2}$, which are CPT-conserving, and $Re\delta$, $Im\delta$, $\frac{Re\mathrm{B}_0}{Re\mathrm{A}_0}$, $\frac{Re\mathrm{B}_2}{Re\mathrm{A}_0}$ and $y$, which are CPT-violating.

The CPLEAR experiment, with its strangeness-tagging capacity in the production and decay of neutral kaons, changes all of this. As listed in Table 2, the CPLEAR experiment allows, in principle, direct determination of all symmetry-violation parameters. The most important additional information comes from the measurement of $Im\delta$, obtained by combining the CP-violation parameters of the main neutral-kaon decays and the measurement of $Re\delta$,

Table 2: The current results on discrete symmetries from the CPLEAR analysis. After completion of the analysis, the precision on the quantities ‡ will be improved by a factor of two.

| Parameter | Symmetry | CPLEAR analysis | Comments |
|---|---|---|---|
| $\Delta m$ | | $(530.1 \pm 1.1)10^7\hbar\mathrm{s}^{-1}$ | Eq. 28 |
| $\varphi_{SW}$ | | $(43.46 \pm 0.08)^o$ | Eq. 28 |
| $Rex$ | | $(8.5 \pm 10.2)10^{-3}$ | ‡ |
| $Imx$ | CP | $(0.5 \pm 2.5)10^{-3}$ | ‡ |
| $\mathrm{A}_T$ | T and CPT | $(6.3 \pm 2.8)10^{-3}$ | ‡ |
| $Re\varepsilon$ | T | $(1.65 \pm 0.01)10^{-3}$ | Eq. 31 |
| $Re\varepsilon_S$ | T and CPT | $(1.65 \pm 0.71)10^{-3}$ | ‡ |
| $\varphi_{+-}$ | CP | $(43.6 \pm 0.6)^o$ | Ref. [24] and Eq. 28 |
| $\varphi_{+-} - \varphi_{SW}$ | CPT | $(0.1 \pm 0.6)^o$ | Eq. 28 |
| $(\varepsilon' - \mathrm{a}\omega)_\perp$ | CPT | $(-0.40 \pm 1.56)10^{-5}$ | Eq. 33 |
| $(\varepsilon' - \mathrm{a}\omega)_\parallel$ | CP and CPT | $(5.41 \pm 1.55)10^{-6}$ | Eq. 34 |
| $Im\mathrm{A}_2/Re\mathrm{A}_0$ | CP | $(7.45 \pm 3.53)10^{-6}$ | ‡ |
| $Re\mathrm{B}_0/Re\mathrm{A}_0$ | CPT | $(-0.14 \pm 9.90)10^{-4}$ | ‡ |
| $Re\mathrm{B}_2/Re\mathrm{A}_0$ | CPT | $(-0.61 \pm 4.96)10^{-5}$ | ‡ |
| $Re\delta$ | CPT | $(-0.1 \pm 9.9)10^{-4}$ | ‡ |
| $Im\delta$ | CPT | $(-0.4 \pm 1.9)10^{-5}$ | Eq. 31 |
| $Rey$ | CPT | $(-0.1 \pm 1.4)10^{-3}$ | ‡ |
| $\lvert\Gamma_{22} - \Gamma_{11}\rvert$ | CPT | $(0.02 \pm 1.46)10^{-17}$ GeV | Eq. 41 |
| $\lvert\mathrm{M}_{22} - \mathrm{M}_{11}\rvert$ | CPT | $(0.04 \pm 6.91)10^{-18}$ GeV | Eq. 41 |
| $\lvert\mathrm{M}_{22} - \mathrm{M}_{11}\rvert$ | CPT | $(0.56 \pm 2.65)10^{-19}$ GeV | assuming $\Gamma_{11} = \Gamma_{22}$ |
| $\alpha$ | CPT | $(-0.5 \pm 2.8)10^{-17}$ GeV | ‡ |
| $\beta$ | CPT | $(2.5 \pm 2.3)10^{-19}$ GeV | ‡ |
| $\gamma$ | CPT | $(1.1 \pm 2.5)10^{-21}$ GeV | ‡ |

obtained from the semileptonic asymmetries. This allows the separate determination of all the CPT-violation parameters and unambiguously probes the $\mathrm{K}^0$–$\overline{\mathrm{K}}^0$ mass and width differences to the order of $10^{-19}$ GeV.

Moreover, the precision of the CPLEAR measurements allows us to probe for the first time quantities, the parameters $\alpha$, $\beta$, $\gamma$ of Table 1, approaching the range of the Planck mass, relevant to effects of quantum gravity [9].

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