MEMORANDUM

18 January 1996

TO: CPLEAR collaboration

FROM: T. Ruf

SUBJECT: About exporting the $\overline{\mathrm{K}}^0/\mathrm{K}^0$ normalization measured in decays to 2π

The normalization measured using the data of $\mathrm{K} \to \pi^+\pi^-$ decays is given by the product of the $\overline{\mathrm{K}}^0/\mathrm{K}^0$ relative tagging efficiency ξ times $\{1 + 4\Re(\varepsilon_\mathrm{L})\}$. Interesting for other decay channels ($\mathrm{K} \to 3\pi$ and semileptonic) is to know the relative tagging efficiency itself. The two semileptonic asymmetries measuring directly T- and CPT-violation have no or practically no decay time dependence. Therefore the absolute knowledge of the $\overline{\mathrm{K}}^0/\mathrm{K}^0$ relative tagging efficiency is of uttermost importance.

One obvious solution to this problem is to take the value of $\delta_l = 2\Re(\varepsilon_\mathrm{L})$ measured by other experiments. This is though not a consistent and recommended procedure for the following reason:

The deviation of the ratio of decays $\overline{\mathrm{K}}^0 \to 2\pi/\mathrm{K}^0 \to 2\pi$ from unity at $\tau = 0$ is, apart from the relative tagging efficiency, only influenced by direct CP violation. This is without any detailed calculations obvious. The factor $\{1 + 4\Re(\varepsilon_\mathrm{L})\}$ is needed to ensure just this:

$$\begin{aligned}
\frac{\overline{R}}{R} &= \xi\,\{1 + 4\Re(\varepsilon_\mathrm{L})\} \cdot \{1 - 4\Re(\eta_{+-}\} \\
&= \{1 + 4\Re(\varepsilon_\mathrm{T} + \delta) - 4\Re(\varepsilon_\mathrm{T} + \delta_\perp + \varepsilon' + i\phi_0)\} \\
&= 1 - 4\Re(\varepsilon') + 4\Re(\delta - \delta_\perp) \qquad (1)
\end{aligned}$$

The difference of $(\delta - \delta_\perp)$ is due to direct CPT violation ($\Gamma_{\mathrm{K}^0} \neq \Gamma_{\overline{\mathrm{K}}^0}$). At $\tau = 0$, one would therefore measure the relative tagging efficiency with a systematic error given by the knowledge of direct CP violation ($\varepsilon' < \mathcal{O}(10^{-6})$ and neglecting direct CPT violation), which is much smaller than the statistical error of the relative tagging efficiency. Using $\Re(\varepsilon_\mathrm{L})$ measured by other experiments and η_{+-} measured by our or other experiments does not automatically fulfill the constraint: $\Re(\varepsilon_\mathrm{L} - \eta_{+-}) = -\Re(\varepsilon')$!

In reality we don't measure the normalization at $\tau = 0$, but in the interval $[1,4]\,\tau_\mathrm{S}$. Therefore one has to correct for indirect CP violation which becomes important at $\tau > 0$. This should be done using: $\Re(\varepsilon_\mathrm{L}) = \Re(\eta_{+-}) + \mathcal{O}(\mathrm{direct\ CP-violation})$

$$\xi = \frac{\int_1^4 d\tau\,\overline{R}(\tau)}{\int_1^4 d\tau\,R(\tau)d\tau}\,\{1 - 4|\eta_{+-}|\cos(\varphi_{+-})\}\,\frac{\int_1^4 d\tau\,\epsilon(\tau)\left(e^{-\tau/\tau_\mathrm{S}} + 2|\eta_{+-}|\cos(\Delta m\tau - \varphi_{+-})\right)}{\int_1^4 d\tau\,\epsilon(\tau)\left(e^{-\tau/\tau_\mathrm{S}} - 2|\eta_{+-}|\cos(\Delta m\tau - \varphi_{+-})\right)} \qquad (2)$$

where $\epsilon(\tau)$ is the average event reconstruction efficiency as function of the decay time. The values of $|\eta_{+-}|$ and φ_{+-} of the $\pi^+\pi^-$ analysis should be used for consistency. The CP correction factor κ_{CP} is about 0.984 and its dependence on η_{+-} is given by $\Delta\kappa_{CP} = -6.6 \cdot \Delta\eta_{+-}$ and on φ_{+-} by $\Delta\kappa_{CP} = -2.2 \times 10^{-4}/1^\circ \cdot \Delta\varphi_{+-}$. These values have been determined assuming a constant efficiency. The error of ξ has to be determined taking into account the correlation between η_{+-} and φ_{+-} (errors for η_{+-} and φ_{+-} are for constant α) :

$$\begin{aligned}
\sigma(\xi)^2 &= \sigma(\frac{\overline{R}}{R})^2 + \left(-6.6, -2.2 \times 10^{-4}\right) \begin{pmatrix} \sigma^2_{\eta_{+-}} & c_{\eta_{+-}\varphi_{+-}} \\ c_{\eta_{+-}\varphi_{+-}} & \sigma^2_{\varphi_{+-}} \end{pmatrix} \\
&= \sigma(\frac{\overline{R}}{R})^2 + 1.3 \times 10^{-7} \\
&\rightarrow \sigma(\xi) \approx 7.7 \times 10^{-4}
\end{aligned}$$

If the normalization would be independent of the primary kaon and pion momenta, one could use the value of α given by the fit to the 2π CP-asymmetry and calculate the relative tagging efficiency using the relation $\xi = \alpha \cdot \{1 - |\eta_{+-}|\cos(\varphi_{+-})\}$ directly. The statistical error of ξ would be given by:

$$\begin{aligned}
\sigma(\xi)^2 &= \begin{pmatrix} 1 - |\eta_{+-}|\cos(\varphi_{+-}) \\ -\alpha|\eta_{+-}|\cos(\varphi_{+-}) \\ \alpha|\eta_{+-}|\sin(\varphi_{+-}) \end{pmatrix}^T \begin{pmatrix} \sigma^2_\alpha & c_{\alpha\eta_{+-}} & c_{\alpha\varphi_{+-}} \\ c_{\alpha\eta_{+-}} & \sigma^2_{\eta_{+-}} & c_{\eta_{+-}\varphi_{+-}} \\ c_{\alpha\varphi_{+-}} & c_{\eta_{+-}\varphi_{+-}} & \sigma^2_{\varphi_{+-}} \end{pmatrix} \cdot \begin{pmatrix} 1 - |\eta_{+-}|\cos(\varphi_{+-}) \\ -\alpha|\eta_{+-}|\cos(\varphi_{+-}) \\ \alpha|\eta_{+-}|\sin(\varphi_{+-}) \end{pmatrix} \\
\sigma(\xi) &= 7.6 \times 10^{-4} \text{ (using values of } 2\pi \text{ paper}
\end{aligned}$$

Ignoring any correlations would give $\sigma(\xi) = 7.1 \times 10^{-4}$. The main contribution to the error of ξ is the statistical error of the 2π normalization value of 6.8×10^{-4}. Therefore we can use the 2π data alone to get a good estimate of the relative tagging efficiency even without assuming CPT !