MEMORANDUM

16 December 1995

TO: CPLEAR collaboration

FROM: T. Ruf

SUBJECT: Ratio of Poisson Distributions

Given two random numbers n and $\overline{n}$ with mean values μ and $\overline{\mu}$, which are distributed as following:

$$f(n) = \frac{\mu^n e^{-\mu}}{n!} \text{ and } \overline{f}(\overline{n}) = \frac{\overline{\mu}\,^{\overline{n}} e^{-\overline{\mu}}}{\overline{n}!} \tag{1}$$

Given the two measured numbers n and $\overline{n}$, what is the best estimate for $\overline{\mu}/\mu$?

First guess: $r = \dfrac{\overline{n}}{n}$

$$\langle r \rangle \;=\; \sum_{\overline{n},n} \frac{\overline{n}}{n} \frac{\mu^n e^{-\mu}}{n!} \frac{\overline{\mu}\,^{\overline{n}} e^{-\overline{\mu}}}{\overline{n}!} \approx \frac{\overline{\mu}}{\mu}\left(1 + \frac{1}{\mu}\right) \text{ for } \mu \gtrsim 50$$

Even with an average of 100 events, the ratio of the two distributions is systematically overestimated by 1% ! The precision required for several CPLEAR results is 10^{-3}.

Correct guess: $r = \dfrac{\overline{n}}{n+1}$

$$\langle r \rangle \;=\; \sum_{\overline{n},n} \frac{\overline{n}}{n+1} \frac{\mu^n e^{-\mu}}{n!} \frac{\overline{\mu}\,^{\overline{n}} e^{-\overline{\mu}}}{\overline{n}!} = \frac{\overline{\mu}}{\mu}\left(1 - e^{-\mu}\right) \tag{2}$$

For $\mu > 10$, $\frac{\overline{n}}{n+1}$ is a good estimate for $\frac{\overline{\mu}}{\mu}$ within the precision required for all CPLEAR results.

Warning: It seems not a good procedure constructing LUT's by ignoring cells with less than a minimum number of events. It is much better to design the cell sizes such that cells with small number of entries are avoided.

Recommendation: One should correct both K^0 and $\overline{K}^0$ with the corresponding LUT, and check for consistency.